PERGAMON

International Journal of Heat and Mass Transfer 42 (1999) 855–860

International Journal of HEAT and MASS TRANSFER

Wavy, wavelike, diffusive thermal responses of finite rigid slabs to high-speed heating of laser-pulses

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Received 12 January 1998; in final form 6 July 1998

Abstract

A generalized macroscopic model is introduced in treating the transient heat conduction problems in finite rigid slabs irradiated by short pulse lasers. The analytical solution is derived by using Green's function method and finite integral transform technique. Various behaviors of conduction heat transfer, such as wave, wavelike, and diffusion, are exhibited by adjusting the relaxation parameters. Then detailed discussions have been given on the interrelations between these behaviors. The calculated temperature responses by this model are compared with two literature results measured under extremely low temperature and ultra-high speed heating, respectively. The calculations show good agreement with the experimental data. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

a thermal diffusivity
c_p specific heat capacity
I output intensity of laser
L thickness
q heat flux
Q absorbed energy density
$\mathbf{r}$ spatial vector
R reflectivity of irradiated surface
t time
T temperature
x spatial variable.

Greek symbols

δ absorption depth of laser irradiation
$\Delta\eta$ dimensionless absorption depth, $\delta/2\sqrt{a\tau_q}$
η dimensionless spatial variable, $x/2\sqrt{a\tau_q}$
θ dimensionless temperature, $\lambda\sqrt{\tau_q}\sqrt{\pi}[T(x,t)-T_0]/(1-R)I_0\sqrt{a}$
λ thermal conductivity
ξ dimensionless time, $t/2\tau_q$, and ratio of relaxation times, $\tau_T/2\tau_q$
ρ density
τ relaxation time.

Subscripts

L thickness
p pulse
q heat flux
T temperature and temperature gradient
0 initial value.

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1. Introduction

The development of high-intense and ultra-short lasers has opened some exciting research opportunities in the field of heat transfer [1–4]. In treating high-speed laser heating problems, two different kinds of models have been frequently employed in previous works. One is the macroscopic thermal wave model, which was postulated by Cattaneo [5] and Vernotte [6], leading to a hyperbolic heat conduction equation and suggesting a finite speed propagation of heat. The other includes those from microscopic point of view, which derived in different forms for various kinds of materials. The microscopic two-step model [7] and the pure phonon field model [8] were developed for evaluating the thermal transport in metals and dielectric solids, respectively, which suggest some behaviors of heat conduction, while neither the macroscopic thermal wave model nor the Fourier thermal diffusion model can do.

To provide a macroscopic description that can capture

0017–9310/98/$—see front matter
PII: S0017-9310(98)00244-0

these microscopic effects, Tzou [9] developed a dual-phase-lag model by generalizing the Fourier's law in 1995. In this model, the microstructural effects are lumped into the delayed response in time, two relaxation parameters were introduced to describe the microscopic interactions. A generalized form of heat flux equation was established, which could predict all the heat conduction behaviors in the microscopic models. Almost the same equation, a heat-flux equation of Jeffreys type, was given by Joseph and Preziosi [2] in 1989 by applying some ideas for describing shear waves in liquids. Such a generalized law makes it possible that the practical engineers may describe various heat transport behaviors by adjusting the relaxation parameters in single governing equation. Inversely, they can determine the relaxation parameters by fitting the solution of this generalized equation to transient temperature measurements, and furthermore type of heat conduction by comparing the relaxation parameters.

The present work considers transient heat conduction in a finite medium exposed to a short laser pulse by introducing the generalized macroscopic conduction model. The analytical solution of the generalized heat conduction equation is derived by using Green's function method and finite integral transform technique. Calculations are performed to exhibit the various type of thermal transports in the medium, such as wave, wave-like, and diffusion. Especially, the wavelike behavior is discussed in detail by comparing it with the wavy and the diffusive ones. To show the adequacy of the present solution, the calculated temperature responses are compared with two literature results measured under extremely low temperature and ultra-high speed heating, respectively. The calculations show good agreement with the experimental data.

2. Generalized macroscopic heat conduction model

Almost the same form of generalized heat flux equation has been derived by Joseph and Preziosi [2] and Tzou [9] from different macroscopic points of view.

Tzou [9] introduces phase lags to the Fourier's law, gives a dual-phase-lag model as

$$\boldsymbol{q}(\boldsymbol{r},t)+\tau_q\frac{\partial \boldsymbol{q}}{\partial t}=-\lambda\left[\nabla T(\boldsymbol{r},t)+\tau_T\frac{\partial}{\partial t}(\nabla T)\right] \tag{1}$$

where, τ_q and τ_T are the phase lags (the relaxation time) of heat flux and temperature gradient, respectively, which are the macroscopic description to the microscopic effects. Applying some ideas of Jeffreys model for the shear waves in liquids to the description of heat flux propagation in solids, Joseph and Preziosi [2] give the same form of heat flux equation as equation (1) but with some parameters from theory of liquids. They describe the microstructural effects by a relaxation function (heat-flux kernel), and decompose it into a fast and a slow mode, which are corresponding to two relaxation times, respectively. Comparing the Jeffreys type of heat flux equation with Tzou's equation shows that the relaxation time of fast and slow mode are corresponding to τ_q and τ_T, respectively. However, for Jeffreys' presentation, the time of fast mode should be smaller than that of slow mode, while for Tzou's presentation, either τ_q or τ_T may be bigger.

Equation (1) and the conservation equation without heat source lead to a description of transient heat conduction in the following generalized equation,

$$\frac{1}{a}\frac{\partial T}{\partial t}+\frac{\tau_q}{a}\frac{\partial^2}{\partial t^2}=\nabla^2 T+\tau_T\frac{\partial}{\partial t}(\nabla^2 T). \tag{2}$$

Comparing equation (2) with microscopic models shows that, if the two relaxation times are formulated properly by some microscopic quantities for different materials, this macroscopic model gives exactly the same heat conduction equation as those in microscopic models. The comparisons can also provide the microscopic physical pictures to the two new introduced relaxation times. For dielectric crystals, from the pure phonon field model [8], the microscopic heat conduction equation has been derived by Joseph and Preziosi [2] and Tzou [9], by comparing with this equation, it can be found that τ_q is the relaxation time for the momentum-nonconserving processes and τ_T has the same order of the relaxation time for the normal processes conserving the momentum in the phonon system. For metals, by comparing with the microscopic two-step model [7], it is found that τ_q captures the relaxation behavior of electron thermal wave conduction, while τ_T captures the effect of phonon–electron interaction. The Fourier's thermal diffusion law and the Cattaneo–Vernotte's thermal wave law are two special cases of this generalized model for $\tau_T=\tau_q=0$ and $\tau_T=0$, respectively [9]. In this work, the generalized model is applied to the heat conduction problem under pulse laser heating and some detail discussions are given about the behaviors predicted by this model.

3. Analysis of one-dimensional problem under laser-pulse heating

Consider a finite rigid slab of thickness L with an initial temperature distribution $T(x,0)=T_0$ (x is the direction along the thickness), constant thermal properties, and insulated boundaries. From $t=t_0$ its front surface ($x=0$) is irradiated uniformly by a laser pulse with Gaussian temporal profile as follows,

$$I(t)=\frac{I_0}{\sqrt{\pi}t_p}\exp\left[-\left(\frac{t}{t_p}\right)^2\right] \tag{3}$$

where t_p is the characteristic time of the laser pulse and I_0 is the laser intensity which is defined as total energy carried by a laser pulse per unit cross-section of the laser beam. According to ref. [10], the conduction heat transfer in the slab can be modeled as a one-dimensional problem with an energy source $Q(x, t)$ near the surface,

$$Q(x,t) = \frac{(1-R)I_0}{t_p \delta \sqrt{\pi}} \exp\left(-\frac{x}{\delta} - \left(\frac{t}{t_p}\right)^2\right) \quad (4)$$

where δ is a characteristic transparent length of irradiated photons called the absorption depth and R is the reflectivity of the irradiated surface. Then the conservation equation of energy can be given by

$$Q(x,t) - \frac{\partial q}{\partial x} = c_p \rho \frac{\partial T}{\partial t}. \quad (5)$$

Combining equation (5) with equation (1) in one-dimensional form lead to the following governing equation to the problem,

$$\frac{1}{a}\frac{\partial T}{\partial t} + \frac{\tau_q}{a}\frac{\partial^2 T}{\partial t^2} = \frac{\partial^2 T}{\partial x^2} + \tau_T \frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial x^2}\right) + \frac{1}{\lambda}\left[Q(x,t) + \tau_q \frac{\partial Q}{\partial t}\right] \quad (0 \leqslant x \leqslant L, t > t_0). \quad (6)$$

For the considered situation, the boundary and initial conditions are taken as

$$\partial T(x,t)/\partial x|_{x=0} = \partial T(x,t)/\partial x|_{x=L} = 0 \quad (7)$$

$$\partial T(x,t)/\partial x|_{t=t_0} = 0, \quad T(x,t)|_{t=t_0} = T_0. \quad (8)$$

Introducing

$$\eta = x/2\sqrt{a\tau_q}, \quad \eta_L = L/2\sqrt{a\tau_q}, \quad \Delta\eta = \delta/2\sqrt{a\tau_q} \quad (9a)$$

$$\xi = t/2\tau_q, \quad \xi_p = t_p/2\tau_q, \quad \xi_0 = t_0/2\tau_q, \quad \xi_T = \tau_T/2\tau_q \quad (9b)$$

and

$$\theta = \lambda\sqrt{\tau_q}\sqrt{\pi}[T(x,t) - T_0]/(1-R)I_0\sqrt{a} \quad (9c)$$

equations (6)–(8) become

$$\frac{\partial^2\theta}{\partial\xi^2} + 2\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} + \xi_T\frac{\partial^3\theta}{\partial\xi\,\partial\eta^2} + \frac{1}{\Delta\eta\,\xi_p}\left(1 - \frac{\xi}{\xi_p^2}\right) \times \exp\left[-\frac{\eta}{\Delta\eta} - \left(\frac{\xi}{\xi_p}\right)^2\right] \quad (0 \leqslant \eta \leqslant \eta_L, \xi > \xi_0) \quad (10)$$

$$\partial(0,\xi)/\partial\eta = \partial(\eta_L,\xi)/\partial\eta = 0 \quad (11)$$

$$\theta(\eta,\xi_0) = \partial\theta(\eta,\xi_0)/\partial\xi = 0. \quad (12)$$

The problem is solved by using the Green's function method and the finite integral transform technique, the temperature distribution is obtained as follows:

$$\theta(\eta,\xi) = \frac{(1-e^{-\eta_L/\Delta\eta})}{2\eta_L\xi_p^2}\int_{\xi_0}^{\xi}[1-e^{2(\tilde\xi-\xi)}]\left(\xi_p - \frac{\tilde\xi}{\xi_p}\right)e^{-(\tilde\xi/\xi_p)^2}\,d\tilde\xi$$
$$+\begin{cases} \dfrac{2}{\eta_L\xi_p^2}\displaystyle\sum_{m=1}^{N}\left\{\dfrac{1-(-1)^m e^{-\eta_L/\Delta\eta}}{\chi\sqrt{\beta}}\cos\left(\dfrac{m\pi\eta}{\eta_L}\right) \otimes \displaystyle\int_{\xi_0}^{\xi} e^{-\left[1+\frac{1}{2}\xi_T\left(\frac{m\pi}{\eta_L}\right)^2\right](\xi-\tilde\xi)}\sin[\sqrt{\beta}(\xi-\tilde\xi)]\left(\xi_p-\dfrac{\tilde\xi}{\xi_p}\right)e^{-(\tilde\xi/\xi_p)^2}\,d\tilde\xi\right\} & \text{when } \beta > 0 \\ \dfrac{2}{\eta_L\xi_p^2}\displaystyle\sum_{m=N}^{\infty}\left\{\dfrac{1-(-1)^m e^{-\eta_L/\Delta\eta}}{\chi\sqrt{-\beta}}\cos\left(\dfrac{m\pi\eta}{\eta_L}\right) \otimes \displaystyle\int_{\xi_0}^{\xi} e^{-\left[1+\frac{1}{2}\xi_T\left(\frac{m\pi}{\eta_L}\right)^2\right](\xi-\tilde\xi)}\sinh[\sqrt{-\beta}(\xi-\tilde\xi)]\left(\xi_p-\dfrac{\tilde\xi}{\xi_p}\right)e^{-(\tilde\xi/\xi_p)^2}\,d\tilde\xi\right\} & \text{when } \beta < 0 \end{cases} \quad (13)$$

where

$$\chi = 1 + \left(\frac{m\pi\Delta\eta}{\eta_L}\right)^2, \quad \beta = \left(\frac{m\pi}{\eta_L}\right)^2 - \left[1 + \frac{1}{2}\xi_T\left(\frac{m\pi}{\eta_L}\right)^2\right]^2 \quad (14)$$

and N is the point at which β changes from a positive number to a negative one with increasing the value of m.

4. Wavy, wavelike, diffusive behaviors

Utilizing equation (13), numerical computation has been performed in order to display the various behaviors of temperature responses arising from the pulse surface heating on the finite rigid slab.

Figure 1 shows the temperature distribution at various instants in a slab with thickness $\eta_L = 2.0$ and absorption depth $\Delta\eta = 0.05$ irradiated by laser pulse with characteristic duration $\xi_p = 0.1$. The ratio between the relax-

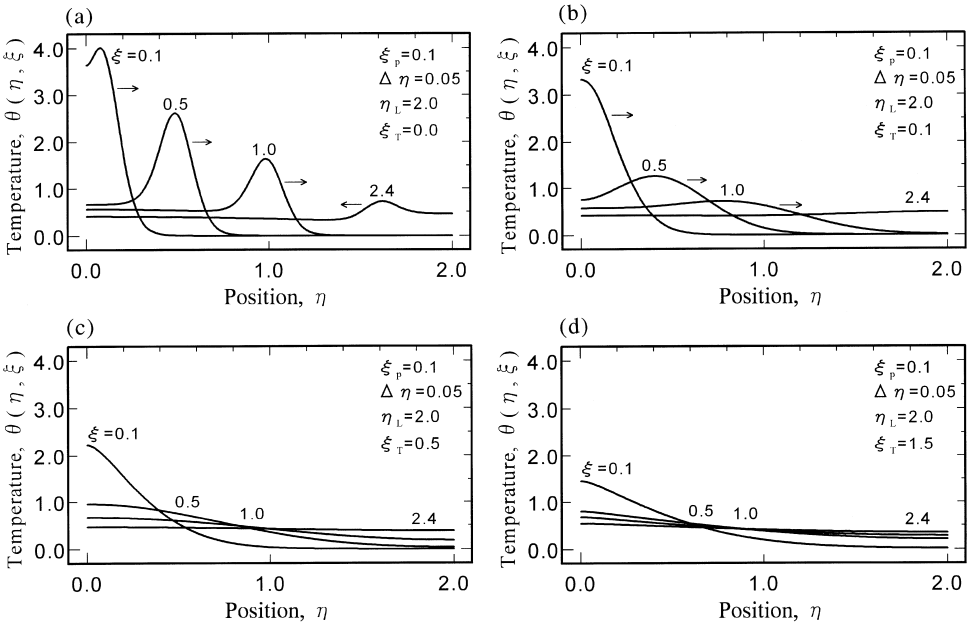

Fig. 1. Temperature distributions in a finite rigid slab for various values of relaxation time. ($\xi_p = 0.1$, $\Delta\eta = 0.05$ and $\eta_L = 2.0$). (a) $\xi_T = 0$ (wave), (b) $\xi_T = 0.1$ (wavelike), (c) $\xi_T = 0.5$ (diffusion), (d) $\xi_T = 1.5$ (over-diffusion).

ation time, $\xi_T = \tau_T/2\tau_q$, is a parameter to control the transition between the different behaviors of heat conduction. In Fig. 1(a), the temperature distributions for $\xi_T = 0$, i.e. $\tau_T = 0$, are plotted for different instants. In this case, the pulse thermal disturbance propagates in the form of wave, and clear wave fronts can be seen at different positions in the figure. Several series of peaks indicate the propagation ($\xi = 0.1$, 0.5, 1.0) and reflection ($\xi = 2.4$) of the temperature wave. Figure 2 shows the temperature responses at both surfaces of a slab with thickness $\eta_L = 1.0$ and absorption depth $\Delta\eta = 0.05$ irradiated by laser pulse with characteristic duration $\xi_p = 0.1$. In the figure, the arrivals of propagated and reflected wave at both surfaces can be seen clearly.

With increasing ξ_T from zero (Fig. 1(b)), the sharp wave fronts are smoothed and the portions of pulse thermal disturbance are dissipated by the diffusive effect of τ_T. For all $\xi_T < 0.5$, i.e. $\tau_T < \tau_q$, some wave features such as propagation and reflection can still be observed. However equation (10) with $\xi_T \neq 0$ is not hyperbolic and it does not give a wave solution. Strictly speaking, the behavior of temperature response for $0 < \xi_T < 0.5$ should be called wavelike behavior, a detail discussion about this behavior will be given later. When $\xi_T = 0.5$ (Fig. 1(c)), all the features of wave disappear, the pulse thermal disturbance transports by diffusion completely. Figure 1(d) shows the temperature behavior in case of $\xi_T > 0.5$, i.e. $\tau_T > \tau_q$, which is called over-diffusion behavior. Comparing with the classical diffusion behavior (Fig. 1(c)), it can be seen that, a bigger τ_T produces high rate thermal diffusion effect and results in rapid temperature response in early time. But it needs a longer time to reach thermal equilibrium than the classical diffusion. These can also be seen from the temperature responses in Fig. 2.

Figure 3 shows the details about the effect of ξ_T. It can be seen that, the increase of ξ_T makes (1) the arrival of wave peaks delayed and the amplitudes attenuated; (2) the edges of wave front smoothed; (3) the portion of pulse thermal disturbance dissipated. When ξ_T is big enough ($\xi_T \geqslant 0.5$), the peak of wave front will disappear completely. The wave features cannot be observed anymore.

Figure 4 shows the comparison between the wavy and wavelike behavior by giving the temperature distributions in a finite slab with thickness $\eta_L = 2.0$ and absorption depth $\Delta\eta = 0.02$ irradiated by a laser with pulse duration $\xi_p = 0.06$. Conceptually a wavy behavior should have the following features: finite propagating speed, wave front, propagation and reflection. For $0 < \xi_T < 0.5$, i.e. $0 < \tau_T < \tau_q$, the temperature responses have almost all the features of wavy behaviors except for the finite propagating speed which offers only the theoretical significance, and in practical engineering problems the wavelike behavior could be absolutely treated as the wavy behavior. Some fine distinctions between these two behaviors, however, should be noticed. By comparing (a) and (b), it can be seen that, the wavelike behavior has smaller amplitude of temperature rise than the wavy one and the increase of ξ_T results in the decrease

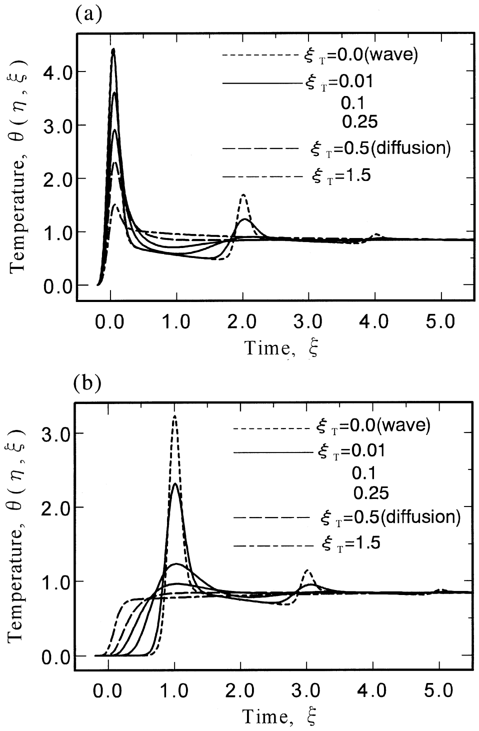

Fig. 2. Temperature responses at the surface of the medium for various values of relaxation time. ($\xi_p = 0.1$, $\Delta\eta = 0.05$, and $\eta_L = 1.0$). (a) Front surface, (b) rear surface.

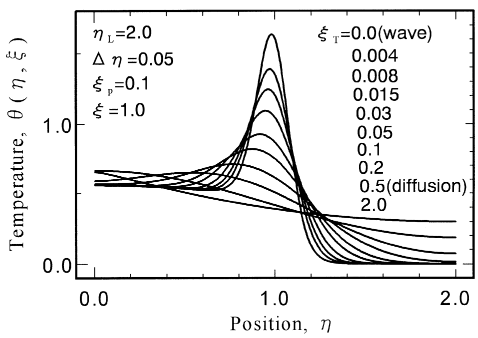

Fig. 3. Comparison of temperature distributions for various values of relaxation time.

of amplitude. For wavy behavior, increasing the propagation distance only results in the attenuation of the amplitude but not any change in the wide of the portion of pulse thermal disturbance, while for the wavelike behavior, the propagation will result in both the attenuation of amplitude and the dissipation of the portion of pulse disturbance.

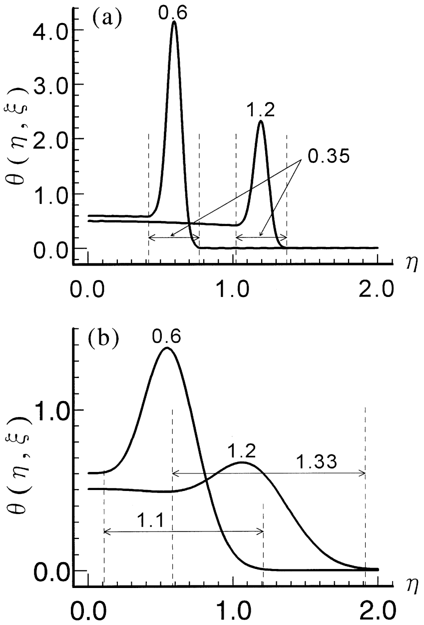

Fig. 4. Comparison between wavy and wavelike behavior. (a) $\xi_T = 0$ (wave), (b) $\xi_T = 0.05$ (wavelike).

5. Comparison with experiments

To show the adequacy of the present analysis, the calculated temperature responses in real coordinate system are compared with two literature results measured under extremely low temperature and ultra-high speed heating, respectively.

Figure 5 shows the temperature response at front surface of a 0.2 μm Au film subjected to 0.096 ps laser-pulse with temporal profile in the form of equation (3), which is measured at room temperature [11]. If the following data of relaxation times and thermal diffusivity from ref. [9],

$$\tau_q = 0.7438 \text{ ps}, \ \tau_T = 89.286 \text{ ps}, \ a = 1.2495 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$$

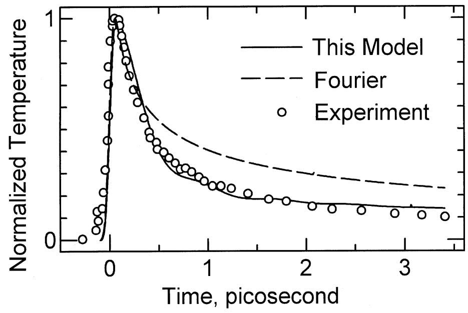

Fig. 5. Temperature response at front surface of Au film subjected to 0.096 ps laser-pulse (at R.T.).

are used as the absorption depth is assumed to be 0.005 μm, the calculation shows very good agreement with the experimental result. By comparing the two relaxation times, it is found that the type of heat conduction is over-diffusion.

Figure 6 shows the temperature response at rear surface of 8 mm solid He4 slab subjected to 5 μs pulse heat flux with temporal profile as follows,

$$I(t) = (I_0 t/t_p^2)\exp(-t/t_p) \tag{15}$$

and the experiment is done at 0.6 K and 54.2 atm [12]. If the absorption depth of this heating flux is δ and the energy is absorbed completely by the slab, the heat source term in equation (6) can be written as

$$Q(x,t) = \frac{I_0 t}{t_p^2 \delta}\exp\left(-\frac{x}{\delta} - \frac{t}{t_p}\right). \tag{16}$$

Then the theoretical temperature response in this case of heating can be obtained by solving equations (6)–(8) with the same procedure as that in the other case. If the heating energy is assumed to be absorbed only by the surface ($\delta < 0.1$ μm) and the relaxation times and thermal diffusivity have the following values, respectively,

$$\tau_q = 61.18\ \mu\text{s}, \quad \tau_T < 0.005\ \mu\text{s}, \quad a = 2.04\ \text{m}^2\ \text{s}^{-1}$$

the calculated curve and experimental data agree with each other. The rapider attenuation in the experimental curve may be caused by the heat loss, which is not considered in the analysis. The theoretical estimation of the relaxation times has not been found in literature, while the value of thermal diffusivity is estimated to be $a = 0.064$–3.01 m^2 s^{-1} for Molar volumes from 11.0–20.9 cm^3 mole^{-1}, by using the reference value of thermal conductivity [13] and Debye temperature [14] and Debye's formula of specific heat capacity. Comparing the two relaxation times shows that the heat conduction in this case has wavy characteristics.

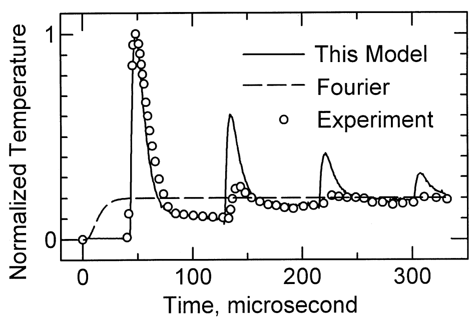

Fig. 6. Temperature response at rear surface of solid He4 subjected to 5 μs heat-pulse (at 0.6 K).

6. Concluding remarks

The analytical solution of the generalized heat conduction equation under the condition of pulse thermal disturbance is derived. Corresponding to the actual situation that a finite rigid slab under pulse laser heating, various behaviors of heat conduction, i.e. the wave, the wavelike, the diffusion and the over-diffusion are obtained by adjusting the relaxation parameters in the generalized heat conduction equation. The temperature responses calculated by this model are compared with two experimental results, and the calculations show good agreement with the experiments. This makes it possible to estimate the relaxation parameters by fitting the solution of this generalized equation to transient temperature responses caused by pulse surface heating, and determine the type of heat conduction by comparing the relaxation parameters in the future work.

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